1 Canonical ensemble of an linear incompressible AB diblock copolymer melt

1.1 Notation and microscopic degrees of freedom

We consider a melt of n identical AB diblock copolymers in a volume V at temperature T. Each chain is described by a space curve $\mathbf{R}_j(s)$ ($j = 1, \dots, n$) parametrized by the contour variable $s \in [0, N]$, where

- N is the total number of segments (monomers) per chain,
- f is the A-block fraction, so $s \in [0, fN]$ is type A and $s \in [fN, N]$ is type B,
- b is the statistical segment length,
- the mean total segment density is

  $$\rho_0 \;=\; \frac{nN}{V} \;=\; \frac{1}{v_0}, \tag{1}$$

  with v_0 the (reference) segment volume.
- $\mathbf{R}^{nN}$ is the short notation of $[\mathbf{R}_1(s), \mathbf{R}_2(s), ...\mathbf{R}_n(s)]$

The canonical partition function (including the standard $n!$ and thermal wavelength prefactor) is written as

$$Z_c(n, V, T) = \frac{1}{n!\,(\lambda_T^3)^{nN}} \prod_{j=1}^{n} \int \mathcal{D}\mathbf{R}_j \;\exp\!\Big[-\beta U_0\big[\mathbf{R}^{nN}\big] - \beta U_1\big[\mathbf{R}^{nN}\big]\Big]\; \delta\big[\hat{\rho}_A + \hat{\rho}_B - \rho_0\big], \tag{2}$$

where $\beta = (k_B T)^{-1}$ and $\delta[\cdots]$ is a functional delta enforcing incompressibility. Here $\hat{\rho}_A, \hat{\rho}_B$ are the microscopic (operator-valued) segment densities defined below.

1.2 Step 1: Chain model (continuous Gaussian chain)

For the continuous Gaussian chain model,

$$\beta U_0\big[\mathbf{R}^{nN}\big] = \frac{3}{2b^2} \sum_{j=1}^{n} \int_0^N \left| \frac{d\mathbf{R}_j(s)}{ds} \right|^2 ds. \tag{3}$$

1.3 Step 2: Flory–Huggins interaction and contact potential

Assume (as in your notes) that A segments interact only with B segments, through a pair potential $u(|\mathbf{r} - \mathbf{r}'|)$. Then

$$\beta U_1\big[\mathbf{R}^{nN}\big] = \sum_{j=1}^{n} \sum_{k=1}^{n} \int_0^{fN} ds \int_{fN}^{N} ds'\; u\big(\big|\mathbf{R}_j(s) - \mathbf{R}_k(s')\big|\big)\,. \tag{4}$$

Choose the standard contact form

$$u(r) \;=\; v_0\, \chi_{AB}\, \delta(r), \tag{5}$$

so the interaction becomes local in space. Introduce the microscopic densities

$$\hat{\rho}_A(\mathbf{r}) \equiv \sum_{j=1}^{n} \int_0^{fN} ds\; \delta\big(\mathbf{r} - \mathbf{R}_j(s)\big), \tag{6}$$

$$\hat{\rho}_B(\mathbf{r}) \equiv \sum_{j=1}^{n} \int_{fN}^{N} ds\; \delta\big(\mathbf{r} - \mathbf{R}_j(s)\big). \tag{7}$$

Using the contact form (5), Eq. (4) reduces to

$$\beta U_1\big[\mathbf{R}^{nN}\big] = v_0\,\chi_{AB} \int d\mathbf{r}\ \hat{\rho}_A(\mathbf{r})\,\hat{\rho}_B(\mathbf{r}). \tag{8}$$

1.4 Step 3: Insert identities to introduce collective density fields

We now insert functional identities (one for each species density):

$$1 = \int \mathcal{D}\rho_A\ \delta\big[\rho_A - \hat{\rho}_A\big], \qquad 1 = \int \mathcal{D}\rho_B\ \delta\big[\rho_B - \hat{\rho}_B\big]. \tag{9}$$

Inserting (9) into (2) gives

$$\begin{aligned} Z_c(n,V,T) = \frac{1}{n!\,(\lambda_T^3)^{nN}} \prod_{j=1}^{n} \int \mathcal{D}\mathbf{R}_j \int \mathcal{D}\rho_A \int \mathcal{D}\rho_B\ \exp\!\Big[-\beta U_0 - \beta U_1\Big] \\ \times\, \delta\big[\hat{\rho}_A + \hat{\rho}_B - \rho_0\big]\ \delta\big[\rho_A - \hat{\rho}_A\big]\ \delta\big[\rho_B - \hat{\rho}_B\big]. \end{aligned} \tag{10}$$

1.5 Step 4: Use the delta constraints

Because $\delta[\rho_A - \hat{\rho}_A]$ and $\delta[\rho_B - \hat{\rho}_B]$ enforce $\rho_\alpha = \hat{\rho}_\alpha$ inside the functional integral, we may replace

$$\delta\big[\hat{\rho}_A + \hat{\rho}_B - \rho_0\big]\ \to\ \delta\big[\rho_A + \rho_B - \rho_0\big], \qquad v_0\chi_{AB} \int d\mathbf{r}\ \hat{\rho}_A\hat{\rho}_B\ \to\ v_0\chi_{AB} \int d\mathbf{r}\,\rho_A\rho_B. \tag{11}$$

Then (10) becomes

$$\begin{aligned} Z_c(n,V,T) = \frac{1}{n!\,(\lambda_T^3)^{nN}} \prod_{j=1}^{n} \int \mathcal{D}\mathbf{R}_j \int \mathcal{D}\rho_A \int \mathcal{D}\rho_B\ \exp\big[-\beta U_0\big[\mathbf{R}^{nN}\big]\big] \exp\Big[-v_0\chi_{AB} \int d\mathbf{r}\,\rho_A\rho_B\Big] \\ \times\, \delta\big[\rho_A + \rho_B - \rho_0\big]\ \delta\big[\rho_A - \hat{\rho}_A\big]\ \delta\big[\rho_B - \hat{\rho}_B\big]. \end{aligned} \tag{12}$$

1.6 Step 5: Fourier representations of the functional deltas (introduce new fields)

Use the (formal) Fourier representation of a functional delta:

$$\delta[F]\ =\ \int \mathcal{D}w\ \exp\!\Big(i \int d\mathbf{r}\ w(\mathbf{r})\,F(\mathbf{r})\Big)\,. \tag{13}$$

Apply (13) to each constraint:

$$\delta\big[\rho_A - \hat{\rho}_A\big] = \int \mathcal{D}w_A\ \exp\!\Big(i \int d\mathbf{r}\ w_A(\mathbf{r})\,[\rho_A(\mathbf{r}) - \hat{\rho}_A(\mathbf{r})]\Big)\,, \tag{14}$$

$$\delta\big[\rho_B - \hat{\rho}_B\big] = \int \mathcal{D}w_B\ \exp\!\Big(i \int d\mathbf{r}\ w_B(\mathbf{r})\,[\rho_B(\mathbf{r}) - \hat{\rho}_B(\mathbf{r})]\Big)\,, \tag{15}$$

$$\delta\big[\rho_A + \rho_B - \rho_0\big] = \int \mathcal{D}\Xi\ \exp\!\Big(i \int d\mathbf{r}\ \Xi(\mathbf{r})\,[\rho_A(\mathbf{r}) + \rho_B(\mathbf{r}) - \rho_0]\Big)\,. \tag{16}$$

Substitute into (12) and group terms into (i) polymer-coordinate dependent pieces (containing $\hat{\rho}_A, \hat{\rho}_B$) and (ii) purely collective-field pieces:

$$\begin{aligned} Z_c(n,V,T) = \frac{1}{n!\,(\lambda_T^3)^{nN}} \prod_{j=1}^{n} \int \mathcal{D}\mathbf{R}_j\ \exp\big[-\beta U_0\big[\mathbf{R}^{nN}\big]\big]\ \exp\!\Big(-i \int d\mathbf{r}\ w_A\,\hat{\rho}_A - i \int d\mathbf{r}\ w_B\,\hat{\rho}_B\Big) \\ \times \int \mathcal{D}\rho_A\mathcal{D}\rho_B\mathcal{D}w_A\mathcal{D}w_B\mathcal{D}\Xi\ \exp\Big[-v_0\chi_{AB} \int d\mathbf{r}\,\rho_A\rho_B\Big] \\ \times \exp\Big[i \int d\mathbf{r}\ \Big(w_A\rho_A + w_B\rho_B + \Xi(\rho_A + \rho_B - \rho_0)\Big)\Big]\,. \end{aligned} \tag{17}$$

1.7 Step 6: Reduce the polymer-coordinate integrals to a single-chain partition function

First rewrite the microscopic coupling to w_A, w_B explicitly in terms of chain coordinates:

$$\int d\mathbf{r}\; w_A(\mathbf{r})\,\hat{\rho}_A(\mathbf{r}) = \sum_{j=1}^{n} \int_0^{fN} ds\; w_A\big(\mathbf{R}_j(s)\big), \tag{18}$$

$$\int d\mathbf{r}\; w_B(\mathbf{r})\,\hat{\rho}_B(\mathbf{r}) = \sum_{j=1}^{n} \int_{fN}^{N} ds\; w_B\big(\mathbf{R}_j(s)\big). \tag{19}$$

Therefore the polymer-coordinate factor in (17) becomes

$$\prod_{j=1}^{n} \int \mathcal{D}\mathbf{R}_j\; \exp\left[-\frac{3}{2b^2} \int_0^N \left| \frac{d\mathbf{R}_j}{ds} \right|^2 ds - i \int_0^{fN} ds\; w_A(\mathbf{R}_j(s)) - i \int_{fN}^{N} ds\; w_B(\mathbf{R}_j(s)) \right]. \tag{20}$$

Because the chains are independent given the fields, this product factorizes:

$$\prod_{j=1}^{n} Z_{\text{chain}}[iw_A, iw_B] = \big(Z_{\text{chain}}[iw_A, iw_B]\big)^n, \tag{21}$$

where the *single-chain partition function in the external fields* is

$$Z_{\text{chain}}[iw_A, iw_B] \equiv \int \mathcal{D}\mathbf{R}\; \exp\left[-\frac{3}{2b^2} \int_0^N \left| \frac{d\mathbf{R}(s)}{ds} \right|^2 ds - i \int_0^{fN} ds\; w_A(\mathbf{R}(s)) - i \int_{fN}^{N} ds\; w_B(\mathbf{R}(s)) \right]. \tag{22}$$

Also define the ideal-chain (no external fields) partition function

$$Z_0 \equiv \int \mathcal{D}\mathbf{R}\; \exp\left[-\frac{3}{2b^2} \int_0^N \left| \frac{d\mathbf{R}(s)}{ds} \right|^2 ds \right]. \tag{23}$$

Introduce the *normalized* single-chain partition function

$$Q[iw_A, iw_B] \;\equiv\; \frac{Z_{\text{chain}}[iw_A, iw_B]}{Z_0}. \tag{24}$$

Thus, the polymer-coordinate contribution equals $(Z_0\, Q[iw_A, iw_B])^n$.

1.8 Step 7: Rewrite Z_c as a field theory and define the effective Hamiltonian

Insert $(Z_0 Q)^n$ into (17). Then

$$Z_c(n, V, T) = \frac{Z_0^n}{n!\,(\lambda_T^3)^{nN}} \int \mathcal{D}\rho_A \mathcal{D}\rho_B \mathcal{D}w_A \mathcal{D}w_B \mathcal{D}\Xi\; \exp\Big[-\beta H[iw_A, iw_B, \rho_A, \rho_B, \Xi] \Big], \tag{25}$$

with the effective Hamiltonian functional

$$\begin{aligned} \beta H[iw_A, iw_B, \rho_A, \rho_B, \Xi] = {}& v_0 \chi_{AB} \int d\mathbf{r}\; \rho_A(\mathbf{r}) \rho_B(\mathbf{r}) \\ & - i \int d\mathbf{r}\; \left[w_A(\mathbf{r}) \rho_A(\mathbf{r}) + w_B(\mathbf{r}) \rho_B(\mathbf{r}) + \Xi(\mathbf{r})(\rho_A(\mathbf{r}) + \rho_B(\mathbf{r}) - \rho_0) \right] \\ & - n \ln Q[w_A, w_B]. \end{aligned} \tag{26}$$

1.9 Step 8: Mean-field (SCFT) saddle point equations

In SCFT (mean-field), we evaluate (25) by the saddle point of H:

$$\left.\frac{\delta \beta H}{\delta X(\mathbf{r})}\right|_{*} = 0, \qquad X \in \{w_A(\mathbf{r}), w_B(\mathbf{r}), \rho_A(\mathbf{r}), \rho_B(\mathbf{r}), \Xi(\mathbf{r})\}. \tag{27}$$

(a) Variations with respect to w_A, w_B: densities from $\ln Q$. From (26),

$$\frac{\delta \beta H}{\delta w_A(\mathbf{r})} = -i\rho_A(\mathbf{r}) - n\,\frac{\delta \ln Q}{\delta w_A(\mathbf{r})}, \qquad \frac{\delta \beta H}{\delta w_B(\mathbf{r})} = -i\rho_B(\mathbf{r}) - n\,\frac{\delta \ln Q}{\delta w_B(\mathbf{r})}. \tag{28}$$

Setting these to zero yields

$$\rho_A^*(\mathbf{r}) = in\left.\frac{\delta \ln Q}{\delta w_A(\mathbf{r})}\right|_{*}, \qquad \rho_B^*(\mathbf{r}) = in\left.\frac{\delta \ln Q}{\delta w_B(\mathbf{r})}\right|_{*}. \tag{29}$$

To make (29) explicit, introduce the standard chain propagators. Define the forward propagator $q(\mathbf{r}, s)$ as the statistical weight density of a chain segment at position $\mathbf{r}$ at contour length s, in the external fields. It satisfies the modified diffusion equation

$$\frac{\partial q}{\partial s} = \frac{b^2}{6}\nabla^2 q - i\,w_\alpha(\mathbf{r})\,q, \qquad q(\mathbf{r}, 0) = 1, \tag{30}$$

where $\alpha = A$ for $0 \le s \le fN$ and $\alpha = B$ for $fN \le s \le N$. Similarly, define the backward propagator $q^\dagger(\mathbf{r}, s)$ (from the chain end) by

$$\frac{\partial q^\dagger}{\partial s} = \frac{b^2}{6}\nabla^2 q^\dagger - i\,w_\alpha(\mathbf{r})\,q^\dagger, \qquad q^\dagger(\mathbf{r}, 0) = 1, \tag{31}$$

with $\alpha = B$ for $0 \le s \le (1-f)N$ and $\alpha = A$ for $(1-f)N \le s \le N$ (i.e. reverse block order). With these, the normalized single-chain partition function can be written as

$$Q[iw_A, iw_B] = \frac{1}{V}\int d\mathbf{r}\; q(\mathbf{r}, N) = \frac{1}{V}\int d\mathbf{r}\; q^\dagger(\mathbf{r}, N). \tag{32}$$

Then the standard result for the segment densities at the saddle point is

$$\rho_A^*(\mathbf{r}) = \frac{n}{V\,Q[w_A^*, w_B^*]}\int_0^{fN} ds\; q(\mathbf{r}, s; [w_A^*, w_B^*])\; q^\dagger(\mathbf{r}, N-s; [w_A^*, w_B^*]), \tag{33}$$

$$\rho_B^*(\mathbf{r}) = \frac{n}{V\,Q[w_A^*, w_B^*]}\int_{fN}^{N} ds\; q(\mathbf{r}, s; [w_A^*, w_B^*])\; q^\dagger(\mathbf{r}, N-s; [w_A^*, w_B^*]). \tag{34}$$

(These match the $q\,q^+$ expressions in your notes.)

(b) Variation with respect to Ξ: incompressibility. From (26),

$$\frac{\delta \beta H}{\delta \Xi(\mathbf{r})} = -i\big(\rho_A(\mathbf{r}) + \rho_B(\mathbf{r}) - \rho_0\big). \tag{35}$$

Thus the saddle point condition gives the incompressibility constraint:

$$\rho_A^*(\mathbf{r}) + \rho_B^*(\mathbf{r}) = \rho_0. \tag{36}$$

(c) Variations with respect to ρ_A, ρ_B: relations between fields and densities. From (26),

$$\frac{\delta \beta H}{\delta \rho_A(\mathbf{r})} = v_0 \chi_{AB} \rho_B(\mathbf{r}) - i w_A(\mathbf{r}) - i \Xi(\mathbf{r}), \tag{37}$$

$$\frac{\delta \beta H}{\delta \rho_B(\mathbf{r})} = v_0 \chi_{AB} \rho_A(\mathbf{r}) - i w_B(\mathbf{r}) - i \Xi(\mathbf{r}). \tag{38}$$

Setting them to zero yields

$$i w_A^*(\mathbf{r}) = v_0 \chi_{AB} \rho_B^*(\mathbf{r}) - i \Xi^*(\mathbf{r}), \qquad i w_B^*(\mathbf{r}) = v_0 \chi_{AB} \rho_A^*(\mathbf{r}) - i \Xi^*(\mathbf{r}). \tag{39}$$

1.10 Step 9: Real-field form (common SCFT notation)

At the saddle point in an incompressible melt, the combinations $i w_A^*$ and $i w_B^*$ are real fields (while w_A^*, w_B^* themselves are purely imaginary in the Fourier representation). It is convenient to define real, N-scaled fields

$$\mu_A(\mathbf{r}) \equiv i N \, w_A^*(\mathbf{r}), \qquad \mu_B(\mathbf{r}) \equiv i N \, w_B^*(\mathbf{r}), \qquad \xi(\mathbf{r}) \equiv i N \, \Xi^*(\mathbf{r}), \tag{40}$$

so that (39) becomes

$$\frac{\mu_A(\mathbf{r})}{N} = v_0 \chi_{AB} \rho_B^*(\mathbf{r}) + \frac{\xi(\mathbf{r})}{N}, \qquad \frac{\mu_B(\mathbf{r})}{N} = v_0 \chi_{AB} \rho_A^*(\mathbf{r}) + \frac{\xi(\mathbf{r})}{N}. \tag{41}$$

Moreover, Q is typically written as a functional of the real fields:

$$Q[\mu_A, \mu_B] = \frac{1}{V} \int d\mathbf{r} \; q(\mathbf{r}, N; [\mu_A, \mu_B]), \tag{42}$$

where the modified diffusion equation takes the same form as (30) but with $-i w_\alpha(\mathbf{r})$ replaced by $-\mu_\alpha(\mathbf{r})/N$ (piecewise in s by block type).

1.11 Final SCFT equations

The SCFT (mean-field) solution is obtained by solving the coupled set:

$$\text{(I)} \quad \rho_A^*(\mathbf{r}) = \frac{n}{V \, Q[\mu_A, \mu_B]} \int_0^{fN} ds \; q(\mathbf{r}, s) \, q^\dagger(\mathbf{r}, N - s), \tag{43}$$

$$\text{(II)} \quad \rho_B^*(\mathbf{r}) = \frac{n}{V \, Q[\mu_A, \mu_B]} \int_{fN}^{N} ds \; q(\mathbf{r}, s) \, q^\dagger(\mathbf{r}, N - s), \tag{44}$$

$$\text{(III)} \quad \rho_A^*(\mathbf{r}) + \rho_B^*(\mathbf{r}) = \rho_0, \tag{45}$$

$$\text{(IV)} \quad \frac{\mu_A(\mathbf{r})}{N} = v_0 \chi_{AB} \rho_B^*(\mathbf{r}) + \frac{\xi(\mathbf{r})}{N}, \tag{46}$$

$$\text{(V)} \quad \frac{\mu_B(\mathbf{r})}{N} = v_0 \chi_{AB} \rho_A^*(\mathbf{r}) + \frac{\xi(\mathbf{r})}{N}, \tag{47}$$

together with the forward/backward diffusion equations for $q, q^\dagger$ (blockwise in s), and

$$\text{(VI)} \quad Q[\mu_A, \mu_B] = \frac{1}{V} \int d\mathbf{r} \; q(\mathbf{r}, N) = \frac{1}{V} \int d\mathbf{r} \; q^\dagger(\mathbf{r}, N). \tag{48}$$

$$\text{(VII)} \quad \frac{\partial q(\mathbf{r}, s)}{\partial s} = \frac{b^2}{6} \nabla^2 q(\mathbf{r}, s) - \frac{\mu_\alpha(\mathbf{r})}{N} q(\mathbf{r}, s), \qquad q(\mathbf{r}, 0) = 1, \tag{49}$$

$$\text{(VIII)} \quad \frac{\partial q^\dagger(\mathbf{r},s)}{\partial s} = \frac{b^2}{6}\nabla^2 q^\dagger(\mathbf{r},s) - \frac{\mu_\alpha(\mathbf{r})}{N}\, q^\dagger(\mathbf{r},s), \qquad q^\dagger(\mathbf{r},0)=1, \tag{50}$$

Under the mean field condition, the effective Hamiltonoan in (26) is the Hemholtz free energy, can also be shown to be:

$$\begin{aligned}
\beta H &\approx \beta A \\
&= v_0\chi_{AB}\int d\mathbf{r}\ \rho^*_A(\mathbf{r})\rho^*_B(\mathbf{r}) \\
&\quad - \int d\mathbf{r}\ \left[\frac{\mu_A(\mathbf{r})}{N}\rho^*_A(\mathbf{r}) + \frac{\mu_B(\mathbf{r})}{N}(\mathbf{r})\rho^*_B(\mathbf{r}) + \frac{\xi(\mathbf{r})}{N}(\rho^*_A(\mathbf{r}) + \rho^*_B(\mathbf{r}) - \rho_0)\right] \\
&\quad - n\ln Q[w_A, w_B].
\end{aligned} \tag{51}$$

1.12 Further simplification

Let's define volume fraction $\phi_A(\mathbf{r})$ and $\phi_B(\mathbf{r})$

$$\tilde{s} = \frac{s}{N}, \qquad \phi_A(\mathbf{r}) = \rho^*_A(\mathbf{r})v_0 = \frac{\rho^*_A(\mathbf{r})}{\rho_0}, \qquad \phi_B(\mathbf{r}) = \rho^*_B(\mathbf{r})v_0 = \frac{\rho^*_B(\mathbf{r})}{\rho_0} \tag{52}$$

We can further self-consistent field system into:

$$\text{(I)} \quad \phi_A(\mathbf{r}) = \frac{1}{Q[\mu_A,\mu_B]}\int_0^f d\tilde{s}\ q(\mathbf{r},\tilde{s})\, q^\dagger(\mathbf{r},1-\tilde{s}), \tag{53}$$

$$\text{(II)} \quad \phi_B(\mathbf{r}) = \frac{1}{Q[\mu_A,\mu_B]}\int_f^1 d\tilde{s}\ q(\mathbf{r},\tilde{s})\, q^\dagger(\mathbf{r},1-\tilde{s}), \tag{54}$$

$$\text{(III)} \quad \phi_A(\mathbf{r}) + \phi_B(\mathbf{r}) = 1, \tag{55}$$

$$\text{(IV)} \quad \mu_A(\mathbf{r}) = N\chi_{AB}\phi_B(\mathbf{r}) + \xi(\mathbf{r}), \tag{56}$$

$$\text{(V)} \quad \mu_B(\mathbf{r}) = N\chi_{AB}\phi_A(\mathbf{r}) + \xi(\mathbf{r}), \tag{57}$$

$$\text{(VI)} \quad Q[\mu_A,\mu_B] = \frac{1}{V}\int d\mathbf{r}\ q(\mathbf{r},1) = \frac{1}{V}\int d\mathbf{r}\ q^\dagger(\mathbf{r},1), \tag{58}$$

$$\text{(VII)} \quad \frac{\partial q(\mathbf{r},\tilde{s})}{\partial \tilde{s}} = \frac{Nb^2}{6}\nabla^2 q(\mathbf{r},\tilde{s}) - \mu_\alpha(\mathbf{r})\, q(\mathbf{r},\tilde{s}), \qquad q(\mathbf{r},0)=1, \tag{59}$$

$$\text{(VIII)} \quad \frac{\partial q^\dagger(\mathbf{r},\tilde{s})}{\partial \tilde{s}} = \frac{Nb^2}{6}\nabla^2 q^\dagger(\mathbf{r},\tilde{s}) - \mu_\alpha(\mathbf{r})\, q^\dagger(\mathbf{r},\tilde{s}), \qquad q^\dagger(\mathbf{r},0)=1. \tag{60}$$

and the intrinsic property of free energy per chain can be written as :

$$\begin{aligned}
\frac{\beta H}{n} &\approx \frac{\beta A}{n} \\
&= \frac{1}{V}\int d\mathbf{r}\ N\chi_{AB}\phi_A(\mathbf{r})\phi_B(\mathbf{r}) \\
&\quad - \frac{1}{V}\int d\mathbf{r}\ \left[\mu_A(\mathbf{r})\phi_A(\mathbf{r}) + \mu_B(\mathbf{r})(\mathbf{r})\phi_B(\mathbf{r}) + \xi(\mathbf{r})(\phi_A(\mathbf{r}) + \phi_B(\mathbf{r}) - 1)\right] \\
&\quad - \ln Q[w_A, w_B].
\end{aligned} \tag{61}$$

The functional in Eq. (61) represents the intensive free energy per chain (in units of k_BT) within the mean-field approximation. Its physical interpretation is decomposed as follows: Physically, this free energy functional per chain decomposes into three distinct contributions:

- **Interaction Enthalpy:** The first integral, $\frac{1}{V}\int d\mathbf{r}\, N\chi_{AB}\phi_A(\mathbf{r})\phi_B(\mathbf{r})$, represents the standard mean-field Flory–Huggins interaction energy. This term provides the thermodynamic drive for microphase separation by penalizing A-B contacts.

- **Field Coupling and Incompressibility:** The second integral subtracts the potential energy coupled to the auxiliary fields. This arises from the Legendre transformation relating the density-based internal energy to the field-based free energy. Specifically, the term involving $\xi(\mathbf{r})$ enforces the local incompressibility constraint $\phi_A(\mathbf{r}) + \phi_B(\mathbf{r}) = 1$ at the saddle point.

- **Conformational Entropy:** The term $-\ln Q[\mu_A, \mu_B]$ represents the dimensionless free energy of a single ideal Gaussian chain subjected to the external fields μ_A and μ_B. It encapsulates the loss of conformational entropy (chain stretching) required to satisfy the spatially varying field configurations.